UNIVERSITEIT GENT

FACULTY OF **SCIENCES**

Department of **Mathematics**

Subject: Review of the PhD thesis of Aidyn Kassymov

"***Basic functional and geometric inequalities for the fractional order operators on homogenous Lie groups***"

your reference	**our reference**	**date**
		13 April 2020

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This PhD thesis is devoted to study the research developments at the intersection of three subjects: functional inequalities, fractional calculus and noncommutative analysis in the setting the homogeneous Lie groups.

In Chapter 3, he obtained fractional analogues of Hardy, Sobolev, Hardy-Sobolev, Gagliardo-Nirenberg, Caffarelli-Kohn-Nirenberg inequalities and logarithmic fractional inequalities on homogeneous Lie groups. Also, he established Hardy-Littlewood-Sobolev and Stein-Weiss inequalities for the Riesz potential on homogeneous Lie groups. In addition, he obtained logarithmic Sobolev-Folland-Stein inequality on stratified groups.

In Chapter 4, he studied a question of the reverse functional inequalities. Firstly, he obtained the reverse integral Hardy inequality on metric measure space with parameters $q < 0$ and $p \in (0, 1)$. As consequences, he obtained integral reverse Hardy inequality on on homogeneous Lie groups, hyperbolic space and Cartan-Hadamard manifoldse with parameters $q < 0$ and $p \in (0, 1)$. Also, he showed integral reverse Hardy inequality on metric measure space with parameters $\infty < q \leq p < 0$ and as a consequences he showed reverse integral Hardy inequality on homogeneous Lie groups. Then he obtained the reverse Hardy-Littlewood-Sobolev, Stein-Weiss and improved Stein-Weiss inequalities on homogeneous Lie groups with parametres $q < 0$ and $p \in (0, 1)$. Also, he obtained the reverse Hardy-Littlewood-Sobolev, Stein-Weiss type and improved Stein-Weiss type inequalities with parameters $\infty < q \leq p < 0$, which is even new in Euclidean settings. In addition, he obtained the reverse Hardy, L^p-Sobolev and L^p- Caffarelli-Kohn-Nirenberg inequalities with the radial derivative on homogeneous Lie groups.

In Chapter 5, he showed applications of the functional inequalities in PDE. Firstly, he obtained Lyapunov inequalities for the fractional p-sub-Laplacian equation and systems on homogeneous Lie groups. As a application of Lyapunov's inequality, he gave lower estimate of the first eigenvalue of the fractional p-sub-Laplacian equation and systems on homogeneous Lie groups. Then, he showed existence of the weak solution for the nonlinear equation with the p-sub-Laplacian on the Heisenberg and stratified groups. Also, he showed existence of the weak solution for the nonlinear equation with the fractional sub-Laplacian and Hardy potential on homogeneous Lie groups and multiplicity of the weak solution with first stratum Hardy potential on Heisenberg and stratified groups. Then he discussed blow-up results for heat equation with fractional sub-Laplacian and logarithmic nonlinearity on homogeneous Lie groups and for heat equation with sub-Laplacian and logarithmic nonlinearity on stratified group. Also, he showed blow-up results for viscoelastic equations with sub-Laplacian on stratified groups, heat and wave Rockland equations on graded groups.

In Appendix, he considered one-dimensional functional inequalities in the Euclidean case. Firstly, he obtained fractional Hardy, Poincaré type, Gagliardo-Nirenberg type and Caffarelli-Kohn-Nirenberg inequalities for the fractional order differential operators as Caputo, Riemann-Liouville and Hadamard fractional derivatives. Also, he showed applications of these inequalities. In addition, he showed Lyapunov and Hartman-Wintner-type

inequalities for a fractional partial differential equation with Dirichlet condition, he gave an application of this inequalities for the first eigenvalue and we show de La Vallée Poussin-type inequality for fractional elliptic boundary value problem.

Many results of this dissertation were published in high-ranked journals: Nonlinear Differential Equations and Applications (Scopus, Web of Science, Q1), Integral Transforms and Special Functions (Scopus, Web of Science, Q2), Complex Variables and Elliptic Equations (Scopus, Web of Science, Q2), Bulletin of the Malaysian Mathematical Sciences Society (Scopus, Web of Science, Q2), Complex Analysis and Operator Theory (Scopus, Web of Science, Q2), Journal of Mathematical Sciences (to appear, Scopus, Q3), Advances in Operator Theory (Scopus, Web of Science).

In my opinion, this thesis fulfils requirements for a doctoral dissertation, so I recommend that Mr. Aidyn Kassymov be awarded the PhD degree.

Yours sincerely,

Prof. Dr. Michael Ruzhansky

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